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Empirical Model of Transport and Decay of Wake Vortices Between Parallel Runways

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This article describes a simple empirical model for the transport and decay of aircraft trailing wake vortices between parallel runways in a turbulent atmosphere. It is suitable for applications to both single and closely spaced parallel runways. The model includes the influence of ambient turbulence on the decay rate of the wake vortices as well as the effect of crosswind shear. Well-known Monin–Obukhoff similarity laws in the lower atmospheric boundary layer are used to compute the ambient shear and turbulence. Vortex bounce that occurs near the ground is also parameterized in the model. The results show that ambient shear plays an important role in vortex decay and transport. Atmospheric stability also affects the fate of these vortices, but with stable stratification having a much larger effect than convective heating of the atmospheric boundary layer. These results may prove useful for a better understanding of the factors that affect the transport and decay of aircraft wake vortices near the ground in the presence of crosswinds in a turbulent atmosphere.

Nomenclature

$a_{1,2}$ = empirical constants in Monin–Obukhov theory
$a_{3,4}$ = constants in vortex transport equations
b = wingspan
$c_{1,2,3}$ = constants in vortex decay equation
g = gravitational acceleration
H = height of the wake vortex center above the ground
M_a = mass of the aircraft
Q = buoyancy flux at the ground
q_* = turbulence velocity scale
R = radial distance from vortex center
R_m = radius of the vortex core defined as the radius of maximum swirl velocity
S_r = spacing between parallel runways
t = time or age of the vortex
$U(z)$ = wind velocity
u = horizontal velocity of the vortex core
u_* = friction velocity
V_a = landing (takeoff) speed of the aircraft
v = swirl (tangential) velocity of the vortex
v_m = maximum swirl (tangential) velocity of the vortex
W_r = width of the runway
w = vertical velocity of the vortex core
w_* = turbulent free-convection velocity scale
X = horizontal position of the wake vortex center
x = coordinate perpendicular to the runway
y = coordinate parallel to the runway
z = vertical coordinate
z_0 = roughness scale
α = angle the wind makes with runway axis
Γ_0 = total circulation around the wake vortex $\Gamma/2\Pi$
ρ = density of air

Subscripts
c = effect of convection from heating of the ground
i = induced velocity
s = effect of wind shear
0 = initial conditions

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Introduction

STRONG vortices trailing behind aircraft are an unavoidable consequence of the generation of the lift needed to sustain the aircraft in flight. These vortices constitute a potential hazard to other aircraft flying in their wake, especially when a small aircraft encounters an intense wake vortex shed by a jumbo transport such as a Boeing 747 or an Air Bus 330. With the advent of the high-capacity/long-haul Boeing 747 jumbo transport aircraft in the early 1970s, the safety hazard became severe enough for aviation authorities to begin to pay serious attention to the problem.[1] Not only were programs initiated to observe, measure, and understand the characteristics, transport, and decay of wake vortices,[1] but potential means of alleviating the hazard were investigated as well.[2] However, no practical and efficient means of accelerating the decay of these vortices has yet been found, leaving flight operations to coexist with these intense vortices in the best way possible.

The wake vortex hazard is particularly serious during the landing and takeoff phases of flight operations, when the proximity of the ground does not allow enough time for recovery in case an encounter should occur. This has led to the establishment of rule-of-thumb separation standards that are strictly enforced at airports, leading to restrictions on flight operations. A serious consequence is a severe restriction on the capacity of busy airports around the world, leading to inefficiencies, congestion, delays, and unnecessary waste of fuel, with the attendant costs to the public and environment.

There has been a phenomenal growth in air traffic around the world in the past decade or so. However, the enormous cost of building modern airports and the public opposition to the construction of new airports, based on environmental and noise pollution considerations, have forced existing airports to accommodate the resulting increase in the number of flight operations as best as they can. The problem is particularly severe at major airports during peak traffic hours and periods. There is therefore a need for more flexible, yet absolutely safe separation standards that would permit safe and efficient flight operations at existing airports. An accurate diagnosis of the wake vortex position and strength as a function of time might permit air traffic controllers to make use of this information to ease some of the restrictions on flight operations. The diagnosis could be based on actual measurements of wake vortices using Doppler lidars and/or reliable numerical models of wake vortex transport and decay.

Many busy airports routinely resort to simultaneous flight operations on parallel runways to improve the flow of traffic. Unless the runways are widely separated, there is the added danger of wake vortices from aircraft operating on one runway being transported to the adjacent runway, posing a potential hazard to aircraft operations there. The problem is particularly serious under crosswind conditions, when the wake vortices can be transported more rapidly onto an adjacent runway before they have had a chance to decay. In addition, observations[3] show that most often, a wake vortex, after descending initially while being transported horizontally, shows a tendency to bounce back up, increasing the chance of being lofted onto the path of the aircraft operating simultaneously on an adjacent runway. At airports such as Frankfurt/Main, where the parallel runways are closely spaced, with a separation distance of only 512 m (compared to 1000 m at a modern airport such as the Denver International), vortex bounce could be a serious problem and it is essential to understand its causes and characteristics. The German aviation authorities have instituted an extensive program to measure the transport and decay of wake vortices of various modern aircraft such as the B757 and A320 at Frankfurt/Main, using continuous wave (cw) infrared (IR) Doppler lidars that have proven to be an effective means of detection, measurement, and tracking of wake vortices. Kopp[3] reports on recent measurements of wake vortices from the B747 class and other aircraft at Frankfurt/Main. These form an excellent basis for at least a qualitative assessment of a model of wake vortex transport and decay. It is the goal of this article to construct a simple empirical model of the wake vortex transport and decay, and explore the parameter space to better understand the factors that influence the longevity of vortex wakes.

The modeling philosophy is similar to that of Greene,[4] who constructed an approximate model of the vortex decay in a turbulent atmosphere, but away from the influence of the ground. He showed that the decay rate was a strong function of both the ambient density stratification and turbulence. This finding is consistent with the experimental results of Liu[5] and Tombach.[6] However, proximity of the ground exerts a major influence on vortex decay and transport (e.g., Bilanin et al.[7]), leading to vortex-bounce,[3] vortex-pair tilting,[8] and enhanced decay of one member of the vortex pair.[7] While these processes are quite complex and not very well understood, we have some idea of the factors that affect them. For example, recent studies[7] appear to indicate that the phenomenon of vortex bounce is principally because of the modification of its structure by shear at the ground by the generation of secondary vorticity of an opposite sign. In any case, it is possible to parameterize the resulting vertical velocity as a function of the relevant external parameters. Similarly, the enhanced dissipation because of ambient shear can be parameterized as a function of the ambient shear. The effect of ambient turbulence can be parameterized as well. With the exception of the phenomenon of vortex-pair tilting in ambient shear,[8] the rest of the effects can be at least approximately accounted for. There is still some uncertainty as to exactly how the ambient shear produces the tilting of the vortex oval. For example, Brashears et al.[8] state that weak shear causes the downwind vortex to be higher, while strong shear causes the upwind vortex to be higher. Neither the precise reason for this behavior nor the underlying physical process is clear and therefore we prefer to exclude this effect from the model. The other effects are at least approximately parameterized.

Trailing vortices often decay from bursting[5] or Crow sinusoidal instability.[5,9,10] These modes of decay are influenced by ambient stratification and turbulence as well. In fact, these modes are the primary mechanisms for vortex decay for a trailing vortex pair far away from the influence of the ground. However, during landing and takeoff, the ground effect, including the existence of a strong ambient shear, can play a dominant role in vortex decay. While these mechanisms might very well destroy trailing vortices before they descend close to the ground and begin to drift apart, it is important for safety reasons to compute an upper bound on wake vortex longevity, which excludes these effects. We therefore ignore Crow instability and vortex bursting in the model.

While the effect of ambient stratification on vortex decay has been studied in the free atmosphere,[4,5] the problem has not been fully explored during landing and takeoff phases, in other words, in the vicinity of the ground and in the presence of significant mean shear. The properties of the atmospheric boundary layer (ABL) adjacent to the ground can be reasonably well-characterized by the Monin–Obukhov (M–O) similarity theory,[11,12] and these in turn used to parameterize the effects of ambient crosswind shear and ambient turbulence on vortex decay and transport.

Observations of Wake Vortex Decay and Transport near the Ground

Kopp[3] presents cw Doppler lidar measurements of trailing vortices for B747 and other classes of aircraft at Frankfurt/Main. While these observations consist of more than 1000 landings under a wide range of ambient stratification conditions, it has not been possible to find a definite correlation between ambient stratification and the vortex decay. However, this aspect can be studied in a model.

Two salient aspects emerge from Kopp's results. One is the pronounced vortex-pair tilt often observed that is induced by crosswind shear[4] showing that one vortex is much closer to the ground and has a larger core diameter than the other. The second observation pertains to the vortex bounce that causes the vortices to bounce back up after initially moving downward toward the ground. We will parameterize vortex bounce but ignore the vortex-pair tilt.

Kopp[3] also presents measurements of the maximum swirl velocity in and the self-induced horizontal velocity on the wake vortices as a function of vortex age. While the scatter is large, it is clear that there are definite trends to be discerned. For example, at least in this data set, wake vortices always decay to a nondangerous state in about 120–140 s for all classes of aircraft. This nondangerous state is arbitrarily defined by Kopp as the state when the maximum swirl velocity in the vortex is below 4 m/s (1 m/s ~ 1.94 kn). The decay can however be faster and in as little as 60 s. Clearly, the decay rate can be a function of ambient turbulence levels as well, and it is important to understand this functional relationship.

Simple Empirical Model of the Wake Vortex Decay and Transport

In this article, we will construct a simple model of the transport and decay of wake vortices over parallel runways, with special attention to crosswind conditions and ground effects. The goal is to arrive at the simplest possible empirical model that captures the essentials, albeit approximately, rather than derive a comprehensive one. We will then explore the parameter space to try and understand the factors that influence the wake vortex decay and transport. We will ignore the initial roll-up phase, when the maximum swirl velocity is either increasing or essentially unchanged and concentrate on the decay phase.

The model is intended to complement more sophisticated vortex decay and transport models based on advanced second moment closure and large eddy simulations of turbulence. While these approaches are very valuable in establishing the physical mechanisms and augment the few observations that are now available, simple empirical models have their utility in exploring the parameter space. Therefore, instead of modeling the turbulence dynamics in detail, we rely on scaling arguments, where dimensional analysis is used to relate the needed internal parameters to relevant external parameters. This method has historically enjoyed considerable success in

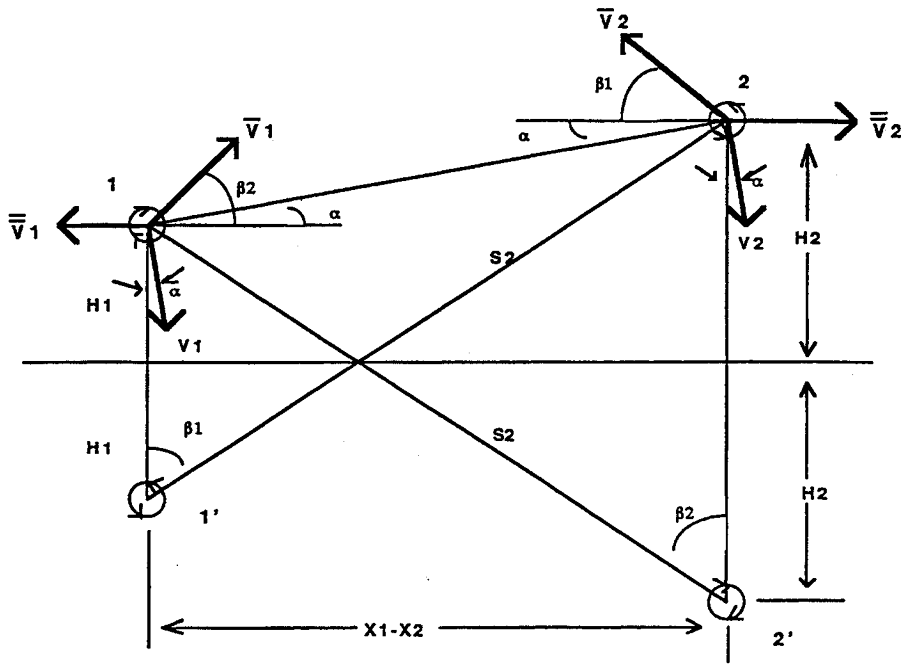

Fig. 1 Sketch of the aircraft trailing vortices near the ground along with their image vortices.

problems involving turbulence dynamics.[11–13] M–O similarity laws and von Kármán universal law of the wall are classic examples.[12,13] The principal drawback is that the underlying physical mechanism must be unambiguously known and the constants in the resulting scaling relationships have to be determined empirically. Ultimately, the success of the scaling approach can only be judged by comparison of the model results with other independent sets of observations. Also, scaling laws are often asymptotic laws, implying large values for governing nondimensional quantities such as Reynolds numbers.

The strength of the trailing vortices shed by an aircraft is a function of the lift on the wing. We will assume that the lift produced is equal to the weight of the aircraft. This is only approximately true, since during landing, the lift is slightly less and during takeoff, slightly more. We will also assume that the circulation around the wake vortices can be obtained by that around the bound vortex on a wing with elliptic lift distribution in a stream with a speed equal to the landing (takeoff) speed of the aircraft. The resulting relationship is accurate to a few per cent:

$$\Gamma_0 = \Gamma/(2\Pi) = M_a g/(0.8\rho b V_a \times 2\Pi) \tag{1}$$

Note that Γ_0 is the conventionally defined circulation Γ divided by the factor 2Π. The swirl velocity in the wake vortex core can be parameterized as[3]

$$v(R) = \Gamma_0 R/(R^2 + R_m^2) \tag{2}$$

where R_m is the radius at which v_m occurs and the circulation is half the total circulation[3]:

$$v_m = \Gamma_0/2R_m \tag{3}$$

The trailing vortex pair is transported vertically down by the velocity induced at the vortex center by the other member of the pair and in close proximity to the ground, by the induced velocity from the image vortices (Fig. 1):

$$w_i = \Gamma_0[(X_2 - X_1)/D_1^2 - (X_2 - X_1)/D_2^2] \tag{4}$$

where

$$D_1^2 = (X_2 - X_1)^2 + (H_2 - H_1)^2 \tag{5a}$$

$$D_2^2 = (X_2 - X_1)^2 + (H_2 + H_1)^2 \tag{5b}$$

Far away from the ground, the induced velocity is primarily vertical, but as the vortex approaches the ground, the image vortices induce a horizontal velocity on the vortices that makes them move away from each other:

$$u_{i,l} = -\Gamma_0[(0.5/H_1) - (H_2 - H_1)/D_1^2 - (H_2 + H_1)/D_2^2] \tag{6a}$$

$$u_{i,r} = +\Gamma_0[(0.5/H_2) + (H_2 - H_1)/D_1^2 - (H_2 + H_1)/D_2^2] \tag{6b}$$

for the left and right vortices, with the observer facing the aircraft. In the presence of a crosswind, the total horizontal velocity is $u_i + U(z)\sin\alpha$.

When crosswind conditions exist, there also exists a crosswind shear, whose magnitude can be written as

$$\frac{dU}{dz} = \frac{u_*}{kz} F_m \sin\alpha \tag{7}$$

where F_m is the M–O similarity function. It is a function of the M–O similarity variable z' that characterizes the influence of stability in the ABL adjacent to the ground. Constant k is the von Kármán constant. $z' = z/L_m$ where L_m is the M–O length scale that accounts for the effect of ambient stratification and is given by

$$L_m = -u_*^3/kQ \tag{8}$$

Positive Q corresponds to heating at the ground, and hence, an unstable ABL and negative values correspond to a stable atmosphere:

$$F_m = (1 + a_1 z') \tag{9}$$

for stable stratification, for example, during nighttime ($a_1 \sim 5$). For example, for unstable conditions during the daytime (especially on a hot summer day),

$$F_m = (1 - a_2 z')^{-1/4} \tag{10}$$

where $a_2 \sim 16$ (Ref. 13).

Relationships (7–10) make use of the fact that in the ABL there exists a layer several tens of meters in thickness adjacent to the ground, where the momentum, heat, and other fluxes are roughly constant and the turning angle of the wind caused by the Earth's rotation can be neglected.[11–14] In this so-called constant flux layer of the atmospheric boundary layer, similarity arguments indicate that the velocity profile can only be a function of the ratio of the height above the ground and the M–O length scale. We assume that these relationships can be used to provide a simple description of the velocity profile over the runways. In practice, the shear profile might be complicated by local terrain and other factors, but these additional factors can be taken into account if need be. Integration of Eq. (7) leads to the familiar log–linear law, which is an extension of the well-known von Kármán logarithmic law of the wall to a stably stratified ABL:

$$U(z)/u_* = (1/k)[\ell n(z/z_0) + a_1 z']\sin\alpha \qquad (11)$$

For an unstable ABL, the corresponding relationship is

$$U(z)/u_* = (1/k)[\ell n(z/z_0) - G]\sin\alpha \qquad (12)$$

where

$$G = 2\,\ell n[0.5(1+f)] + \ell n[0.5(1+f^2)] - 2\tan^{-1}f + \Pi/2 \qquad (13)$$

$$f = 1/F_m = (1 - a_2 z')^{1/4} \qquad (14)$$

In addition, the turbulence velocity scale is a function of stability as well and can be written as[14]

$$q_* = 2.55 u_* (F_m - z')^{1/3} \qquad (15)$$

These properties can be used to parameterize the influence of ambient stability and turbulence on vortex decay.

Many recent studies[7,15,16] have shown that the phenomenon of the vortex bounce near the ground is intimately tied up with the generation of secondary vortex adjacent to the ground in front of the advancing vortex. Thus the shear induced by the presence of no-slip conditions is an important factor. If it is the shearing of the vortex by the ground that causes vortex bounce near the ground,[7] the induced vertical velocity on the vortex can be parameterized as

$$w_s = a_3 \Gamma_0 / H \qquad (16)$$

Constant b_3 is an empirical coefficient chosen to be consistent with observations: $a_3 \sim 0.03$.

When the ground is heated during a hot summer day, there exists a mean upward velocity over the hot runway and a general subsidence over the relatively cooler surface between the runways. The vertical velocity w_c caused by this convection can be parameterized as proportional to Prandtl's convective velocity scale w_* given by

$$w_* = a_4 (Qz)^{1/3} \qquad (17)$$

over the runway ($a_4 \sim 0.2$). Over the ground in between the runways, the vertical velocity is parameterized as

$$w_c = -w_*(W_r/S_r) \qquad (18)$$

The height of the wake vortex above the ground can be written as

$$\frac{dH}{dt} = w_i + w_s + w_c \qquad (19)$$

and its horizontal position is given by

$$\frac{dX}{dt} = u_i + U(z) \qquad (20)$$

Note that the origin is located at the center of the left runway. We will consider only the wake vortices generated over the left runway. Extension to the right runway is quite straightforward.

To complete the set of equations, we need an equation for the rate of change of the maximum swirl velocity with time or equivalently the rate of change of core radius with time. Now, the wake vortex is imbedded in the turbulent atmospheric boundary layer. The ambient turbulence can be characterized by q_* as indicated previously, which is a combination of u_* and w_*. The rate at which the vortex decays is a function of q_*. Larger values of q_* can be expected to accelerate the decay. During daytime, convective turbulence augments shear-generated turbulence. During nighttime, there is stable stratification and the turbulence is mainly confined to the immediate vicinity of the ground and is shear-generated by geostrophic wind aloft.

The maximum swirl velocity in the core is therefore parameterized here as a function of time by:

$$\frac{dv_m}{dt} = -\left(c_1 \frac{\Gamma_0}{r_m^2} + c_2 \frac{q_*}{r_m}\right) v_m \qquad (21)$$

The first term denotes vortex decay in the absence of external influences. The second term corresponds to the acceleration of the decay rate by ambient turbulence. In addition, there are indications[7] that in the presence of ambient shear, the vortex with its vorticity opposite in sign to that of the background vorticity due to wind shear, the downwind vortex, undergoes accelerated decay often leading to the phenomenon of the lone surviving wake vortex. We parameterize this effect by adding a term to the right-hand side of Eq. (21) equal to

$$-c_3 \frac{dU}{dz} v_m \qquad (22)$$

for the downwind vortex only. Empirical constants c_1, c_2, and c_3 have been chosen to be consistent with Kopp's wake vortex observations: $c_1 \sim 0.006$, $c_2 \sim 0.02$, and $c_3 \sim 0.5$. However, there is considerable uncertainty in the values of these constants and more observational data are needed to determine them more accurately.

Equations (19–22) can be integrated from any prescribed initial conditions to determine the position in space and the strength of the wake vortices at any given time. Initially, we assume that the vortices are spaced apart by a distance equal to $0.8b$ and are located symmetrically on either side of the center of the runway at a height of H_0.

Model Results

The results described below are for $H_0 = 60$ m. Extension to other initial heights is straightforward. The initial vortex radius has been assumed to be 3 m, which appears to yield reasonable numbers for the initial values of v_m. For example, for a B747, this yields a value of 18 m/s and for an aircraft with a weight of 140,000 kg, a value of 12.5 m/s for v_m, and these values are consistent with Kopp's data, once the vortices have rolled up fully. It also leads to smaller values of initial v_m for lighter aircraft, which is also consistent with observations. Integrations are carried out for 180 s in each case.

We will present results for a wide variety of modern transport aircraft (B727-200, B737-400, B747-300, B757, B767, B777-200, A310, A330, A300, A600, MD-11, MD-90, C17A, and C5A) at various wind and ambient stability conditions. The characteristics of these aircraft (rounded off) are shown

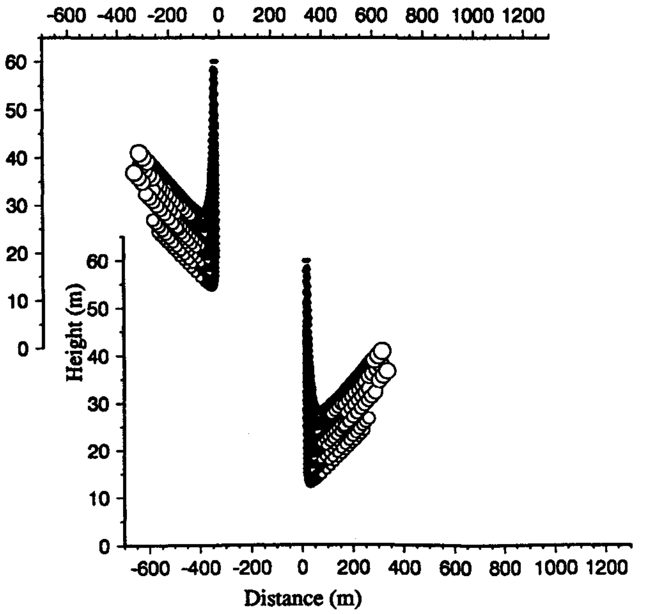

Fig. 2 Position and strength of trailing vortices of the suite of aircraft in Table 1 as they descend from an initial height of 60 m, without any ambient wind and stratification. The vortices are shown staggered vertically and horizontally for clarity. The size of the circle is proportional to the size of the vortex core. Filled circles indicate hazardous and unfilled circles nonhazardous conditions. Note the mirror symmetry about the centerline and the vortex bounce.

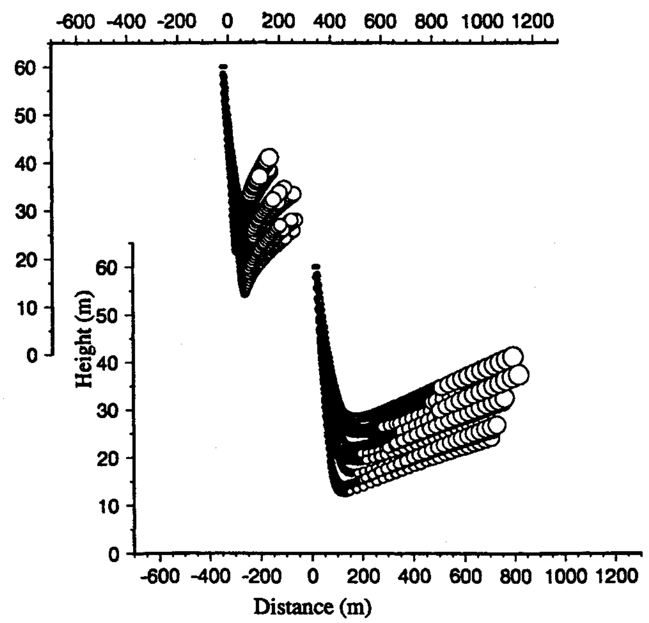

Fig. 3 Position and strength of trailing vortices as they descend from an initial height of 60 m, with 2.5-m/s ambient crosswind and neutral atmospheric stratification. Filled circles indicate hazardous and unfilled circles nonhazardous conditions. Note the rapid lateral transport of the leeward vortex and the stronger bounce of the upwind one.

in Table 1. The crosswind, if any, will be assumed to blow from left to right. The positions of the upwind and downwind vortices will be shown looking head-on towards the landing aircraft, plotted (staggered horizontally and vertically for clarity) every second when the vortex is hazardous but less frequently (every 6–10 s) when it is not. The size of the circle is proportional to the vortex core radius. The symbols are chosen deliberately to highlight the intensity of the vortex; filled circles indicate hazardous levels of the maximum swirl velocity. Unfilled circles denote a nonhazardous condition, which is defined as the condition when the maximum swirl velocity in the vortex has decayed to values below 4 m/s. This definition, while consistent with that used by Kopp,[3] is quite arbitrary. Other definitions are possible and can be accommodated by the model code.

Figure 2 shows the positions of the two vortices at zero wind and neutral stability conditions in the ABL. In the absence of ambient wind and stratification, the vortex decay and transport are purely functions of the vortex characteristics. The decay and motion of the two trailing vortices are similar; there is symmetry about the vortex oval center. The vortices descend

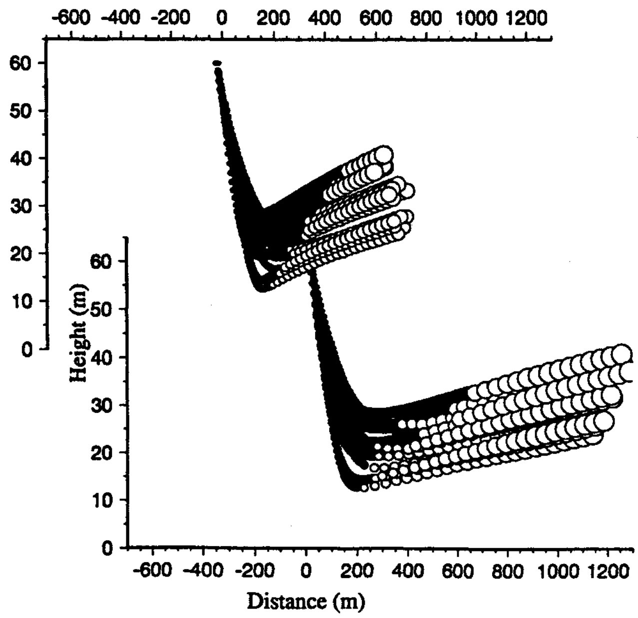

Fig. 4 Position and strength of trailing vortices of the suite of aircraft in Table 1 as they descend from an initial height of 60 m, with 5-m/s ambient crosswind and neutral atmospheric stratification. Filled circles indicate hazardous and unfilled circles nonhazardous conditions. Note the rapid lateral transport of both vortices.

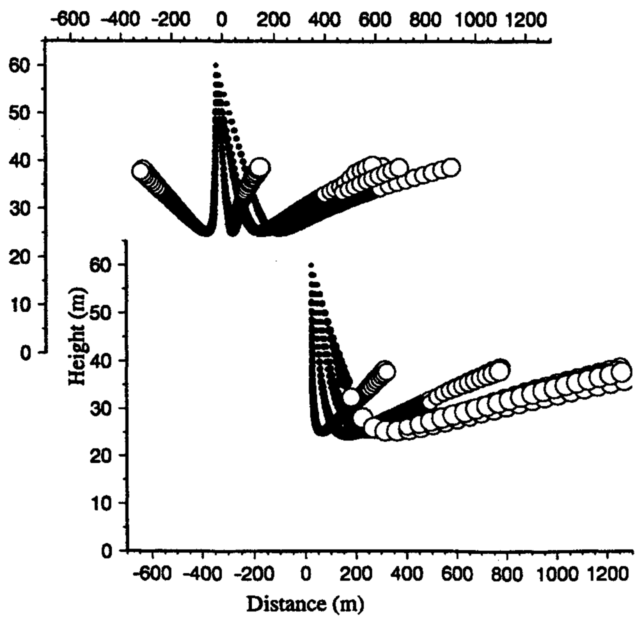

Fig. 5 Position and strength of B747 trailing vortices as they descend from an initial height of 60 m, at 0-, 2.5-, and 5-m/s ambient crosswinds, and neutral, daytime convective and nighttime stable atmospheric stratification conditions. Filled circles indicate hazardous and unfilled circles nonhazardous conditions.

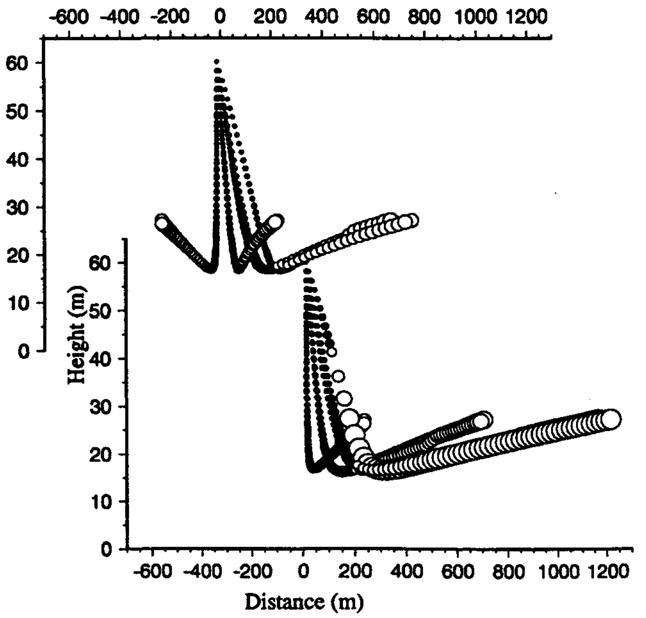

Fig. 6 Position and strength of B757 trailing vortices as they descend from an initial height of 60 m, at 0-, 2.5-, and 5-m/s ambient crosswinds, and neutral, daytime convective and nighttime stable atmospheric stratification conditions. Filled circles indicate hazardous and unfilled circles nonhazardous conditions.

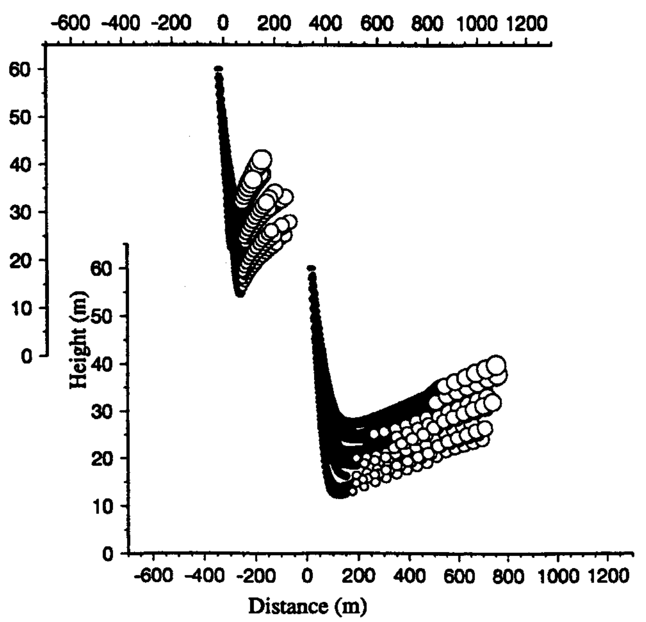

Fig. 7 Position and strength of trailing vortices of the suite of aircraft in Table 1 as they descend from an initial height of 60 m, for 2.5-m/s ambient crosswind and under daytime unstable atmospheric stratification. Filled circles indicate hazardous and unfilled circles nonhazardous conditions. Note the increase in upward motion when the vortices hit the parallel runway.

vertically to a certain height initially, before moving apart horizontally. The smaller the weight of the aircraft generating them, the closer they descend to the ground over the runway, before drifting away from the runway. Therefore, it is the vortices of the smaller aircraft that tend to linger longer right above the runway in the absence of a crosswind. Also, in the proximity of the ground, the vortices begin to move upward as well, soon after they begin to drift apart in the horizontal direction. The degree of this vortex bounce is qualitatively similar to that presented by Kopp. It is also clear that under zero wind conditions, none of the previous vortices may pose a potential hazard to operations on a parallel runway, if it is spaced more than 400 m apart; they decay well enough before they impact the parallel runway, at least according to this model.

Figure 3 shows the model results for a 5-m/s (~10-kn) wind (at the anemometric height of 10 m) blowing at 30 deg to the runway axis, yielding a crosswind velocity of 2.5 m/s (~5 kn). The ABL is assumed to be neutral. Under these conditions, the vortex decay is enhanced by ambient turbulence. The downwind vortex decays much faster than the upwind one because of the influence of ambient crosswind shear. Both vortices are transported perpendicular to the runways by the cross-

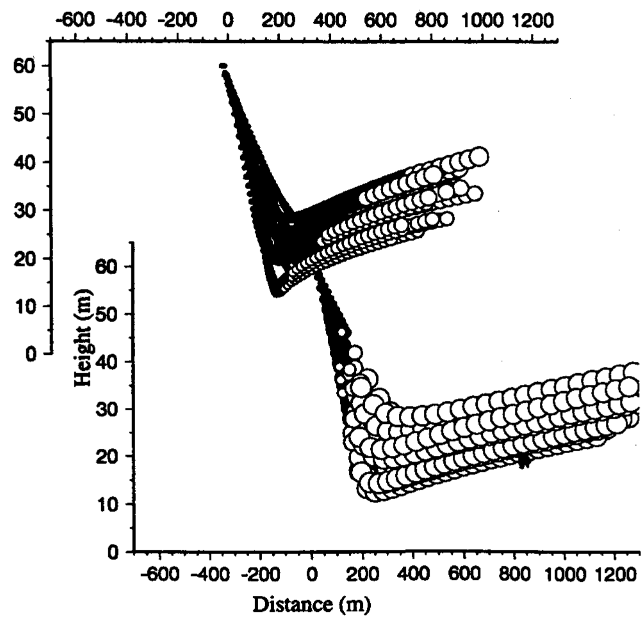

Fig. 8 Position and strength of trailing vortices of the suite of aircraft in Table 1 as they descend from an initial height of 60 m, for 2.5-m/s ambient crosswind under stable nighttime atmospheric stratification. Filled circles indicate hazardous and unfilled circles nonhazardous conditions. Note the strong effect of stable stratification on the downwind vortex.

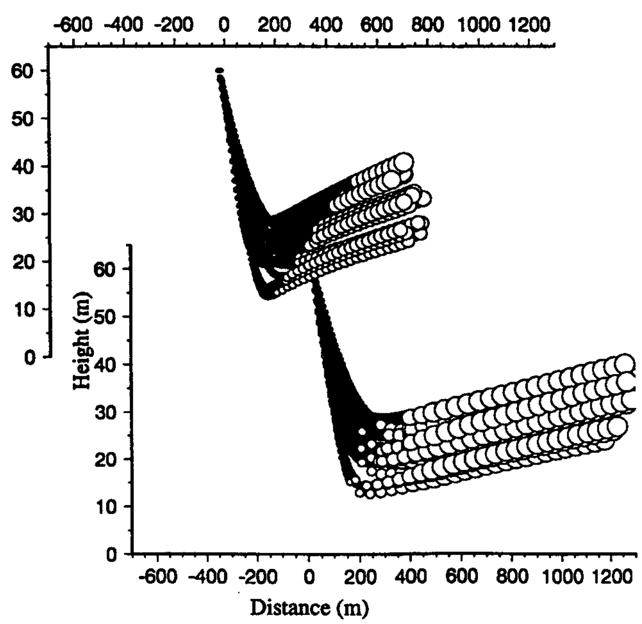

Fig. 9 Position and strength of trailing vortices of the suite of aircraft in Table 1 as they descend from an initial height of 60 m, for 5-m/s ambient crosswind under stable nighttime atmospheric stratification. Filled circles indicate hazardous and unfilled circles nonhazardous conditions. Note the strong effect of stable stratification on the downwind vortex.

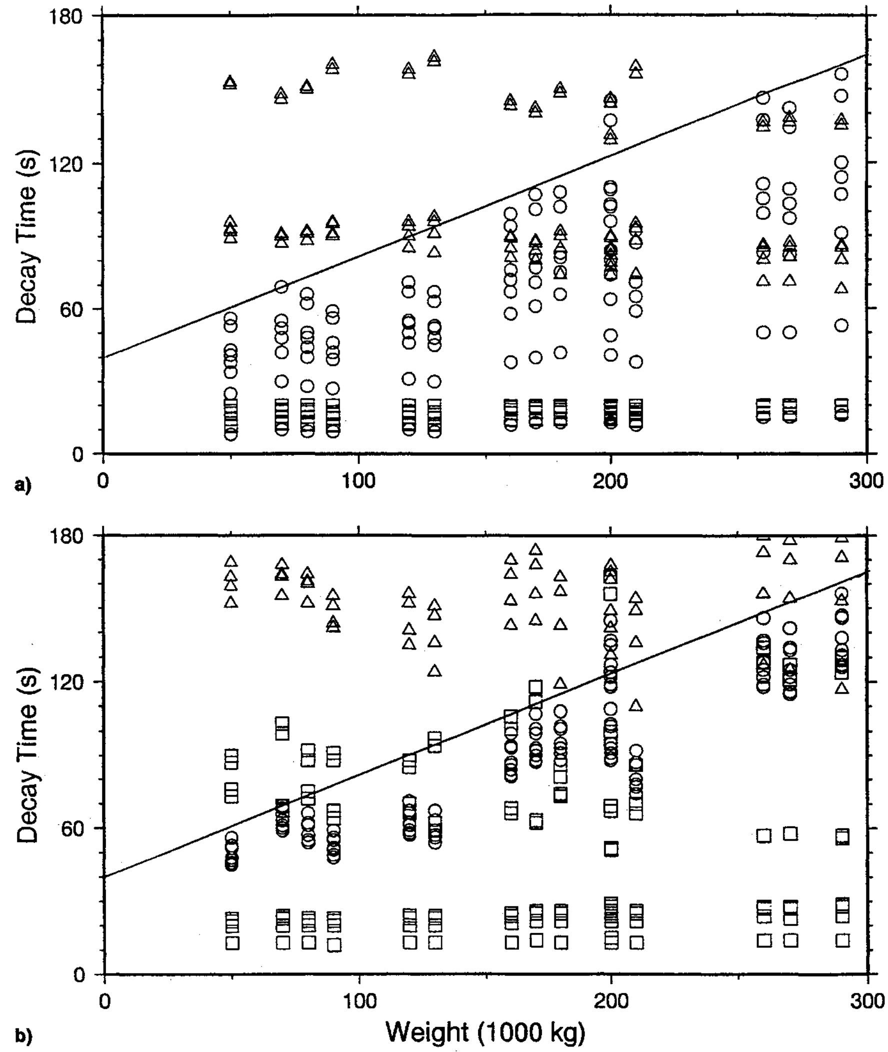

Fig. 10 Various times for the a) downwind and b) upwind vortex as a function of aircraft weight. Squares indicate the time for the vortex to clear the runway. Circles denote the time for the vortex to decay to nonhazardous swirl intensity and triangles the time to impact the parallel runway.

wind, although it is the downwind one that is transported more rapidly, and therefore has the potential for impacting the parallel runway before decaying to a nonhazardous state. However, the results show that except for certain aircraft, both vortices decay to nonhazardous conditions before they reach a parallel runway 500 m away. For a runway more than 700 m away, none of the vortices appear to constitute a potential hazard. The vortex bounce is quite significant for most cases.

Figure 4 pertains to a wind velocity of 10 m/s (~20 kn) at 30 deg. The results are similar to those for the 5-m/s (~10-kn) winds, except that the downwind vortex is transported even faster, and therefore, more of the vortices tend to be potentially hazardous to operations on a parallel runway.

We next describe wake vortex transport and decay for two aircraft, a B747 (weight ~260,000 kg, wingspan ~60 m) and a B757 (weight ~90,000 kg, wingspan ~38 m), under various ambient conditions to explore their effect on vortex decay and transport. Landing speed for both aircraft is assumed to be 60 m/s (~117 kn). Figure 5 shows the results for a B747 for zero wind conditions, and under neutral and daytime convective (with a heat flux from the ground of 100 W/m^2) conditions. Also shown are results for wind velocities of 5 (~10 kn) and 10 m/s (~20 kn) blowing at 30 deg to the runway axis, not only under neutral and unstable ABL conditions as shown earlier, but also for a stable nighttime ABL (with a heat flux of 10 W/m^2 to the ground). It is apparent that none of the vortices descend below a height of 24 m above the ground. This lowest descent height is once again consistent with Kopp's data.[3]

Crosswind introduces a mean wind shear that tends to attenuate the downwind vortex preferentially, once again consistent with observations. However, the downwind vortex is transported rapidly away from the runway onto any existing parallel runway. These are the major differences from zero wind conditions. Surprisingly, the ambient unstable stratification has no major impact. The difference between the convective daytime conditions and neutral conditions is hardly discernible, except when the wake vortex passes over the parallel runway, when convective heating bounces the vortex up. While the vortex decay rate is enhanced somewhat by the increase in the ambient turbulence level brought on by convection, the mean shear remains roughly similar so that there is not a big difference in the decay rate of the two wake vortices for unstable and neutral conditions. Nocturnal stable stratification on the other hand enhances the mean shear considerably so that the downwind vortex decays more rapidly. It is therefore possible that the lone vortex phenomenon is likely to occur more often during nighttime than during the day. These results are consistent at least qualitatively with observations presented by Kopp,[3] from which it is rather difficult to discern any distinct stability effects on vortex decay.

Figure 6 shows the model results for a B757. Apart from the lower descent of its vortices (16 m as opposed to 24 m for

Table 1 Aircraft characteristics

Aircraft	Weight, kilo kg	Span, m	Landing speed, m/s	Γ_0, m^2/s
B727-200	70	33	54	65
B737-400	50	29	53	54
B747-300	260	60	60	118
B757	90	38	68	57
B767	130	48	72	62
B777-200	210	61	71	79
A300	120	44	69	64
A310	160	45	69	84
A330	270	60	63	116
A340-300	180	59	55	91
A600	170	45	69	89
MD-11	200	52	69	91
MD-90	80	33	65	61
C5A	290	68	56	126
C17A	200	50	55	118

a B747), the results are very similar to those for a B747. Note that the B757 vortices tend to descend longer and lower compared to those of a B747, and therefore, tend to loiter longer over the runway.

Figure 7 presents results for the entire suite of aircraft mentioned earlier for a 5-m/s (~10-kn) wind at 30 deg, but for daytime convective conditions. Notice the similarity of the vortex positions to those in Fig. 3, confirming the small influence of unstable stratification. Figures 8 and 9 present results for stable stratification at 5-m/s (~10-kn) and 10-m/s (~20-kn) ambient wind conditions. Stable stratification exerts a higher influence on the downwind vortex. Results for zero wind but stable conditions are not shown, simply because under the absence of ambient shear and turbulence, the vortex decay is similar to that under zero wind and neutral stratification conditions.

Figure 10 shows some salient times as a function of the aircraft weight, for the downwind (top) and upwind (bottom) vortex respectively, for all the ambient conditions considered in this study: 1) the time for the vortex to clear (100 m from runway centerline) the runway (squares), 2) the time for the vortex to decay to 4 m/s maximum swirl velocity (circles), and 3) the time for the vortex to impact the parallel runway (triangles). Several conclusions are possible. For single runway operations, it is the upwind vortex that is more troublesome. While both vortices clear the runway in less than 30 s under zero wind conditions, and strong crosswinds tend to also blow both vortices clear of the runway, it is the light to moderate crosswinds that are hazardous. Under these conditions, the upstream vortex tends to loiter in close vicinity of the runway and it is necessary to wait for it to decay to a nonhazardous state. However, since in most cases, the vortices clear the runway before they decay to nonhazardous conditions, it might be possible to ease the separation requirements, especially for wakes of smaller aircraft. The vortex decay period (circles) itself is roughly a linear function of aircraft weight and this fact could be factored into more flexible separation standards. With the exception of aircraft in the 200,000-kg category, it is possible to fit a straight line through these data so that the minimum safe separation time (according to this model) for all modern transport aircraft listed in Table 1 is given (in seconds) by

$$t_{min} = 40 + 0.43W_t \qquad (23)$$

where W_t is the weight of the aircraft in kilogram (metric ton).

For parallel runway operations, both vortices need to be considered, especially for strong crosswind conditions, even though the downwind vortex is attenuated slightly more rapidly. Knowing the time for decay and the time of potential crossing of each vortex might better delineate a safe window for simultaneous operations. It is, however, the downwind vortex that is more likely to impact the parallel runway, and it is possible once again to fit the same straight line to both the downwind and upwind vortex decay data in Fig. 10 to indicate the minimum time for decay of the trailing vortices. As long as the arrival time of the vortices at the parallel runway is longer than this, simultaneous operations are feasible.

Even a simple empirical model such as this has the potential to ultimately provide useful advisory information needed for more flexible but safe separation rules.

Concluding Remarks

A simple empirical model of the transport and decay of aircraft wake vortices between parallel runways has been formulated. The model accounts for the effects of ambient turbulence on vortex decay. It also includes the effect of ambient crosswinds and crosswind shear on the vortex decay and transport. The vortex bounce observed quite frequently is parameterized as well. While the model results are quite realistic, there is a definite need for detailed comparisons with individual vortex wakes, which are not available from Kopp.[3] Clearly, because of the empirical nature of the model, refinement of the various empirical constants is highly desirable. Once a high level of confidence is achieved as to the correctness of the model and its constants, the model can be tested in a hindcast/forecast mode and compared with in-situ observations from a Doppler lidar. Only when validated thus, can it be useful for operational applications.

An unexpected and surprising model result is that convective heating of the ground appears to only slightly affect the fate of the vortices, while stable stratification exerts more influence. This finding appears to be consistent with recent observations[18,19] of vortex wakes, which could not establish unambiguously the influence of ambient stability. The mean crosswind shear tends to attenuate the downwind vortex more. Crosswinds do tend to transport both vortices onto a parallel runway, which under some conditions could pose a potential hazard to flight operations on a closely spaced parallel runway.

It is important to note that the model ignores the initial roll-up phase of the trailing vortex. This initial phase can be added onto the model by assuming that v_m and R_m remain unchanged for a certain characteristic time scale dependent on the aircraft. From Kopp's[3] observations, this initial phase appears to last typically 30–40 s. For a rough estimate, it should be possible to just add this to the vortex age in the model. The model also assumes that the wake vortices stay intact, and neither the Crow instability nor bursting affects their fate. Therefore, the model estimates are rather conservative. However, we do not know enough about these modes of decay in the vicinity of the ground to accurately parameterize them in the model. Also, while recent studies[7,15,16] suggest, and we have assumed, that the vortex bounce is because of the self interaction of the vortex with the ground, the precise mechanisms are still to be regarded as quite ambiguous. For example, the role of the ambient crosswind shear on the vortex bounce is not yet clear. It is also important to include the vortex pair tilt, once the primary factor causing and controlling the phenomenon is known unambiguously.

Finally, it is worth pointing out that we have assumed the circulation around each vortex of the decaying pair to be constant at the initial value given by Eq. (1). While this is consistent with the desire for conservative estimates as to the danger posed by the wake vortices, more accurate estimates will have to account for the decrease in circulation with the time of each vortex caused by vorticity diffusion from the other. This effect can be readily accommodated by solving Eq. (19) to Eq. (22) along with

$$\frac{d\Gamma_0}{dt} = -c_4 \frac{\Gamma_0^2}{D_1^2} \qquad (24)$$

In deriving Eq. (24), we have assumed that the time scale for the vortex decay caused by diffusion of vorticity from the other member of the pair must be inversely proportional to the square of the distance between them. Since the circulation is the most relevant quantity, this time scale becomes D_1^2/Γ_0. It is to be expected that this effect would be important in the initial phases of descent of the pair, when the vortices are close together. Once they begin to move apart, the effect should decrease rapidly. However, the data needed to determine the constant c_4 are of uncertain reliability and we will not pursue this aspect any further in this article. Similarly, alternative definitions of vortex hazard will be the subject of future work. The principal goal of this article is to demonstrate feasibility, not necessarily to present definitive final results.

A model such as this, when fully tested, calibrated, and validated, might be useful for obtaining rough estimates of the position and strength of wake vortices over single and parallel runways. Information on friction velocity and buoyancy flux can be readily obtained by measuring the velocity and temperature at two discrete heights above the ground, i.e., 10 and 20 m. In conjunction with vortex measuring/tracking instrumentation such as the Doppler lidar, such a model might enable more efficient flight operations to be conducted at major airports around the world.

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